Physics 129a **Homework 2** **Due 10/12/23**

1. Suppose that G is a finite group of order N.

- Show that for any element $g \in G$, the order of g must be a divisor of N. (Hint: consider the subgroup of G generated by g.)
- If the order of a group is a prime number p, show that the group has to be a cyclic group.

2. The center of a group, denoted as $Z(G)$, is the set of elements that commute with every element of G.

$$Z(G) = \{a \in G \mid ag = ga, \forall g \in G\} \tag{1}$$

Consider the Dihedral group D_4 (the symmetry of a square).

- What is the center Z of D_4?
- What is the group D_4/Z?

3. Suppose that H is a subgroup of G. Show that if *the set of* left cosets $\{g_i H | g_i \in G\}$ is the same as *the set of* right cosets $\{Hg_j | g_j \in G\}$, then H is a normal subgroup satisfying $gH = Hg$ for all $g \in G$.

4. Consider the quaternion group $Q = gp\{x, y\}, x^4 = e, x^2 = y^2, y^{-1}xy = x^{-1}$.

- What is the order of the group? (Hint: list all possible distinct compositions of powers of x and powers of y.)
- Decompose Q into conjugacy classes.
- Find one (proper) normal subgroup of Q and the corresponding quotient group.
- Is Q isomorphic to D_4? Explain your answer.