Physics 129a **Homework 3** **Due 10/19/23**

1. Consider the cyclic group C_n.

- If n is the product of two distinct primes p_1 and p_2, is C_n isomorphic to $C_{p_1} \times C_{p_2}$? Explain your answer.
- If n is the square of a prime p, is C_n isomorphic to $C_p \times C_p$? Explain your answer.

2. Given a normal subgroup N of a group G, a representation $D^{G/N}$ of the quotient group G/N can be lifted to give a representation D^G of the full group G by the following definition:

$$D^G(g) := D^{G/N}(gN) \tag{1}$$

That is, each element of the group is assigned the matrix $D^{G/N}$ of the coset to which it belongs. Suppose that $D^{G/N}$ is faithful.

- Verify that $D^G(g)$ indeed provides a representation of G, i.e. $D^G(g_1)D^G(g_2) = D^G(g_1 g_2)$.
- What is the kernel of this representation?

3. Find n different one dimensional representations of the cyclic group C_n. Verify that they are orthogonal to each other.

4. f is a group homomorphism from group A to group B. Follow the steps below and show that $A/\text{kernel}(f) \cong \text{img}(f)$.

- kernel(f) is the set of elements in A that are mapped to the identity element in B: $\text{kernel}(f) = \{a \in G | f(a) = e_B\}$. Show that kernel$(f)$ is a normal subgroup of A. (Please check the group axioms to show that it is a group.)
- img(f) is the set of elements in B that are mapped to from elements in A: $\text{img}(f) = \{b \in B | \ \exists a \in A, \ s.t. f(a) = b\}$. Show that img$(f)$ is a group. (Please check the group axioms to show that it is a group.)
- Show that $A/\text{kernel}(f) \cong \text{img}(f)$. That is, show that there is a one-to-one correspondence between the cosets of kernel(f) and elements in img(f) and that the composition rules match.