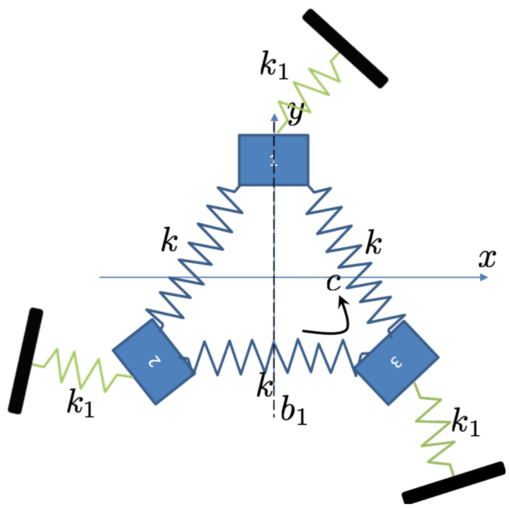
1. Coupled oscillator under perturbation



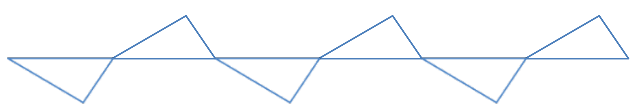
Consider the coupled oscillator as shown in the figure. The blue mass blocks all have the same mass and can move in the 2D xy plane. The black bars represent walls. Consider small oscillations around this equilibrium configuration.

(1) When $k_1 = 0$, the system has full D_3 symmetry. According to what we discussed in class, what conclusion can we draw about the eigenmodes of the system?

(2) When $k_1 \neq 0$, reflection symmetry is broken while rotational symmetry is still preserved. How could the eigenmodes of the system change? (Use symmetry arguments. Do not solve the equation of motion directly)

2. Non-symmorphic space group

Consider the one dimensional pattern as shown in the following figure (extending from left infinity to right infinity). All the triangles in this pattern are of the same shape.



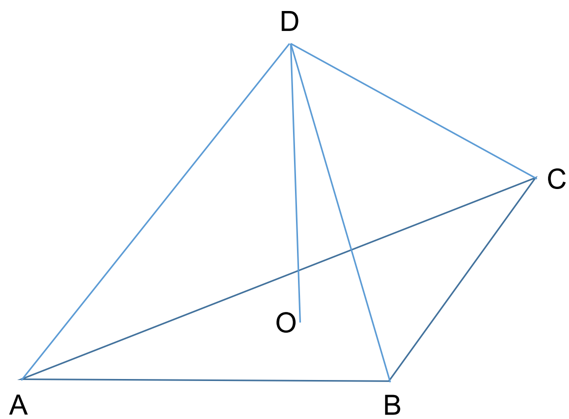
(a) What are the translation operations that keep the pattern invariant?

(b) What are the point group operations that keep the pattern invariant?

(c) Are there symmetry transformations that cannot be obtained by composing the transformations in (a) and (b)?

3. The point group C_{3v} is, among other things, the symmetry group of the ammonia molecule NH_3, which forms a right pyramid on an equilateral triangle base, as shown below. $AB = BC = CA$. O is the center of the triangle and OD is perpendicular to ΔABC.

The symmetry group is generated by a 3-fold rotation c around the axis OD and a reflection σ_v with respect to plane OAD.



(a) Show that $C_{3v} \simeq$ (is isomorphic to)D_3.

(b) Show that the molecule can possess a permanent electric dipole moment $\vec{P}$. What is the direction of this electric dipole moment?

(c) A magnetic dipole moment, like a magnetic field, is an *axial* vector. Under reflection symmetry, it gets an extra minus sign compared to a regular vector (such as electric dipole). For example, under reflection with respect to the xy plane, the magnetic field (M_x, M_y, M_z) goes to $(-M_x, -M_y, M_z)$. Its transformation under rotation is the same as a regular vector. Show that the NH_3 molecule cannot possess a permanent magnetic moment.

4. The tetragonal lattice can be obtained from the cubic lattice by stretching in the z direction such that the lattice constant in the z direction is not equal to that in the x and y direction.

(a) what kind of rotation symmetry does the lattice have?

(b) what kind of reflection symmetry does the lattice have?

(c) Consider a macroscopic property of the material described by a rank two tensor σ_{ij} such that $\sigma_{ij} = \sigma_{ji}$. How many independent degrees of freedom does the tensor have in a material with tetragonal lattice symmetry?