Introduction

What kind of waves have you heard of?

Sound wave, water wave, light wave, electromagnetic wave, seismic wave, gravitational wave, quantum mechanical wave ...

Wave as a form of motion is ubiquitous. Moreover, understanding waves is a key step in going from classical mechanics to quantum mechanics. That is why we are having a whole course on waves.

Syllabus

In this course, we will study the generation, propagation, and various properties of waves.

- Simple harmonic oscillation: one degree of freedom; Forced oscillation and resonance
- Normal modes: simple harmonic oscillation of more than one degrees of freedom
- Waves: free and forced oscillation in an infinite system
- Traveling waves; Standing waves
- Reflection and transmission
- Modulations, pulses, and wave packets
- Waves in two and three dimensions: Polarization
- Interference and Diffraction

Logistics

- Instructor: Xie Chen, Office: 163 W. Bridge, Phone: x3793, Email: xiechen@caltech.edu. Office hour: TBD.
- TA: Ina Soerensen, isorense@caltech.edu; Alexander (Zander) Moss, zander@caltech.edu. Recitation and office hour: TBD.
- Grader: TBD.
- Lectures: Thursdays 10:30 am - 12:00 pm, 201 E Bridge.
- Prerequisites: Math 1abc or equivalent (differential equations, matrices, trigonometry, complex numbers, etc.); Physics 1abc, or equivalent (mechanics, electromagnetism, etc.); Mathematica (basics).

- Recommended textbooks (not required):
  - Georgi, The Physics of Waves (can be downloaded from http://www.people.fas.harvard.edu/ hgeorgi/new.htm)
  - Crawford, Waves (can be downloaded from https://archive.org/details/Waves_371)
  - French, Vibrations and Waves (on reserve at the library)
  - The Feynman Lectures, chap 21-36, 47-51
    (can be viewed online at http://www.feynmanlectures.caltech.edu/)
  - Mathematica training sessions http://url.wolfram.com/2FLjmnPC/
- Problem Sets

  Posted on the website every Thursday, due on the following Thursday. Please scan or take a picture and submit on Canvas. Solutions will be posted also on the website.
- Exams

  Final exam in the last week of the term.
- Grading

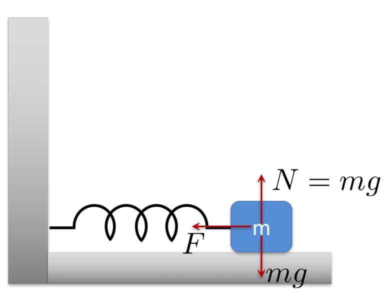
  60% problem sets, 40% final exam.
- Extension policy

  One silver bullet for one week extension (no question asked). Please communicate with grader before using the silver bullet. For extra extensions, please email me.

1 Simple Harmonic Oscillator

To study waves, we start from its most elementary components: a simple harmonic oscillator (SHO). A SHO is a small periodic motion of one degree of freedom. It can be realized in many different forms.

1.1 Example I: mass on a spring



Consider the small oscillation of a mass attached to a massless spring on a frictionless surface. The one degree of freedom is x – the position of the mass. The mass experiences three forces: the gravitational force mg which is balanced by the supporting force from the surface $N = mg$ and the restoring force from the spring F. Therefore, the motion $x(t)$ of the mass is only affected by the restoring force F. Suppose that x_0 is the position of m when the spring is relaxed, then Hooke's law says

$$F = -k(x - x_0) \tag{1}$$

where k is the spring constant. Notice the minus sign in this formula, indicating that the force is a restoring force.

Without loss of generality, let's set $x_0 = 0$. We have $F = -kx$. Combined with Newton's law $F = ma = m\frac{d^2x}{dt^2}$, we get the equation of motion

$$m\frac{d^2x}{dt^2} = -kx \tag{2}$$

To solve this equation, let's try solutions of the form $x(t) = A\cos(\omega t) + B\sin(\omega t)$, which contains three unknown variables A, B and ω. Taking the first derivative with respect to time, we get

$$\frac{dx}{dt} = -A\omega\sin(\omega t) + B\omega\cos(\omega t) \tag{3}$$

Taking derivative again, we get

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t) \tag{4}$$

Plugging this back to the equation of motion we find

$$-m\omega^2(A\cos(\omega t) + B\sin(\omega t)) = -k(A\cos(\omega t) + B\sin(\omega t)) \tag{5}$$

Hence

$$\omega = \sqrt{\frac{k}{m}} \tag{6}$$

ω is called the angular frequency of the oscillation.

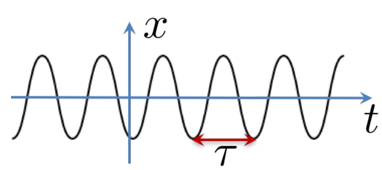
The equation of motion alone cannot determine A and B. They are related to the initial conditions of the oscillation. From the form of the solution we can see that

$$x(t = 0) = A,\ x'(t = 0) = B\omega \tag{7}$$

Therefore, the general form of the solution is given by

$$x(t) = x(0)\cos(\omega t) + \frac{1}{\omega}x'(0)\sin(\omega t) \tag{8}$$

which graphically looks like



The time it takes for the motion to repeat itself is called a period, which we denote by τ, is equal to $\frac{2\pi}{\omega}$. We have

$$x(t + \tau) = x(t),\ x'(t + \tau) = x'(t) \tag{9}$$

The frequency of the oscillation ν is given by $\nu = \frac{1}{\tau}$, which is related to the angular frequency as $\omega = 2\pi\nu$.

We can in general put $x(t)$ into the form

$$x(t) = C\sin(\omega t + \phi) \tag{10}$$

where $C = \sqrt{(x(0))^2 + \frac{(x'(0))^2}{\omega^2}}$ is called the amplitude of the oscillation and ϕ, determined from $\sin\phi = x(0)/C$, $\cos\phi = x'(0)/(\omega C)$, is called the phase of the oscillation.